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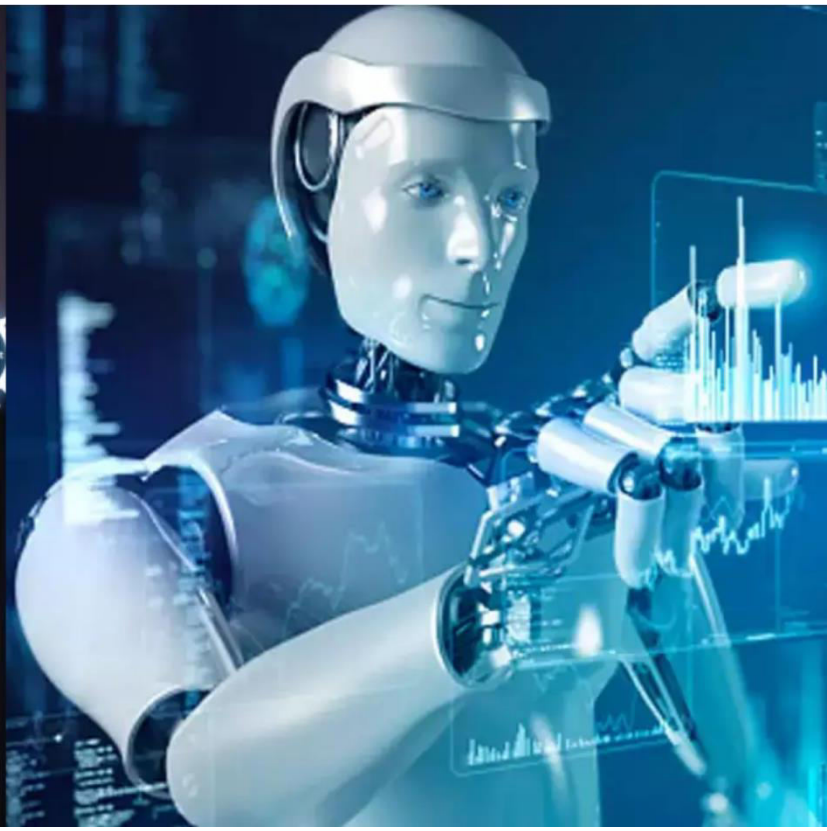

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Two-Dimensional Inverse Transient Thermoelastic Problem of a Solid Sphere

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ABSTRACT: This study examines a two-dimensional inverse transient thermoelastic problem for a homogeneous, isotropic solid sphere. The temperature is prescribed on an interior spherical surface, whereas the temperature distribution on the outer surface is unknown and must be reconstructed. The governing heat-conduction equation is formulated in spherical coordinates with axisymmetric angular dependence. A Legendre-polynomial expansion is combined with the Laplace transform to obtain a modal representation of the temperature field and an expression for the unknown boundary temperature in the transform domain. The analysis also presents displacement, strain, and thermoelastic-stress relations associated with the temperature field. A special angular-temperature input is considered, and illustrative plots of temperature, displacement, and stress distributions are included. The formulation provides an analytical framework for investigating inverse thermal-boundary problems in spherical bodies.

KEYWORDS: Inverse thermoelasticity; transient heat conduction; solid sphere; Laplace transform; Legendre polynomials; spherical Bessel functions; boundary-temperature reconstruction.

I. INTRODUCTION

Inverse thermoelastic problems arise when thermal information is available at an interior location but the boundary temperature or heat flux responsible for that response is unknown. Such problems are relevant when direct measurement of the surface thermal state is difficult, while internal temperature data can be obtained.

Earlier work by Ghonge and Ghadle considered one-dimensional inverse transient thermoelastic problems for solid and hollow spheres under radial symmetry [1, 2]. Related integral-transform approaches were subsequently applied to circular plates [3, 4], including work on inverse thermoelastic behaviour using Marchi–Fasulo and Laplace transforms [5]. Further studies addressed inverse thermoelastic behaviour in circular plates and thermal analysis of annular discs [6, 7]. These studies provide the background for extending the analysis beyond purely radial dependence.

The present paper considers an axisymmetric, two-dimensional setting for a homogeneous isotropic solid sphere. The temperature is known on an interior spherical surface, while the corresponding outer-surface temperature is treated as unknown. The heat equation is separated into angular modes using Legendre polynomials, and the Laplace transform is used to express the relationship between the interior and exterior modal temperatures. Classical treatments of integral transforms, heat conduction, thermal stress, and elasticity provide the broader theoretical context [8–11].

II. FORMULATION OF THE PROBLEM

Consider a homogeneous, isotropic solid sphere of radius a. Let r and θ denote the radial and polar coordinates, respectively, with $0 \le r \le a$ and $0 \le \theta \le \pi$. The temperature field $T(r, \theta, t)$ is assumed to be axisymmetric and independent of the azimuthal coordinate.

$$\frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta} \right) \right] \tag{1.1}$$

The initial temperature is taken to be zero:

$$T(r, \theta, 0) = 0 \tag{1.2}$$

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The prescribed temperature on the interior spherical surface $r = \xi$ is

$$T(\xi,\ \theta,\ t) = f(\theta,\ t) \tag{1.3}$$

The unknown temperature on the outer surface $r = a$ is

$$T(a,\ \theta,\ t) = g(\theta,\ t) \tag{1.4}$$

Here, k denotes thermal diffusivity, a is the outer radius, and ξ is the radius of the interior measurement surface, with $0 < \xi < a$. The functions f and g represent the known interior temperature and the unknown outer-boundary temperature, respectively.

III. DISPLACEMENT AND STRESS RELATIONS

The radial and angular displacement components in terms of the temperature field are given as []

$$u_r = \alpha r T(r,\ \theta,\ t) \tag{1.5}$$

$$u_\theta = \alpha \sin\theta T(r,\ \theta,\ t) \tag{1.6}$$

The corresponding radial, circumferential and shear stresses are:

$$\sigma_{rr} = -\frac{E\alpha T}{(1\ -\ 2\nu)} \tag{1.7}$$

$$\sigma_{\theta\theta} = -\frac{E\alpha T}{(1\ -\ \nu)} \tag{1.8}$$

$$\sigma_{r\theta} = -\mu\alpha T \tag{1.9}$$

Here, α is the coefficient of thermal expansion, E is Young's modulus, ν is Poisson's ratio, and μ is the shear modulus.

IV. SOLUTION OF HEAT CONDUCTION EQUATION

To represent the angular dependence, expand the temperature field in Legendre polynomials:

$$T(r,\theta,t) = \sum_{n=0}^{\infty} T_n(r,t) P_n(\cos\theta) \tag{1.10}$$

Substitution into the heat-conduction equation, together with the Legendre differential equation, gives the following radial-time equation for each mode n

$$\frac{\partial T_n}{\partial t} = k\left[\frac{\partial^2 T_n}{\partial r^2} + \left(\frac{2}{r}\right)\frac{\partial T_n}{\partial r} - n(n\ +\ 1)\frac{T_n}{r^2}\right] \tag{1.11}$$

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Taking the Laplace transform with respect to time and imposing regularity at the centre gives a modal solution in terms of a spherical Bessel function:

$$\overline{T_n}(r,\ s) = A_n\, j_n(qr), \quad q\ = \sqrt{\frac{s}{k}} \tag{1.12}$$

Here, j_n is the spherical Bessel function of the first kind, and A_n is a mode-dependent constant in the transform domain. Let $\overline{f_n}(s)$ denote the Laplace transform of the nth Legendre coefficient of the known interior temperature. Applying the condition at $r = \xi$ gives

$$A_n = \frac{\overline{f_n}(s)}{j_n(q\xi)} \tag{1.13}$$

Evaluating the modal solution at the outer radius $r = a$ yields the relation for the unknown boundary-temperature coefficient:

$$\overline{g_n}(s) = \left[\frac{j_n(qa)}{j_n(q\xi)}\right]\overline{f_n}(s) \tag{1.14}$$

Applying the inverse Laplace transform and summing over all angular modes reconstructs the boundary temperature:

$$g(\theta,t) = \sum_{n=0}^{\infty} P_n(cos\theta) L^{-1}\left[\frac{j(qa)}{j(q\xi)}\overline{f_n}(s)\right] \tag{1.15}$$

Equation (1.15) expresses the outer-surface temperature in terms of the known interior-temperature data and a transfer factor for each Legendre mode.

V. THERMO-ELASTIC ANALYSIS

Once the temperature field has been determined, the displacement components may be expanded in the same angular basis:

$$u_r = \sum_{n=0}^{\infty} U_n(r,\ t) P_n(cos\ \theta) \tag{1.16}$$

$$u_\theta = \sum_{n=0}^{\infty} V_n(r,\ t) \frac{dP_n(cos\ \theta)}{d\theta} \tag{1.17}$$

The radial and circumferential normal strains are written as

$$\varepsilon_{rr}\ = \frac{\partial u_r}{\partial r} \tag{1.18}$$

$$\varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta}\right) + \frac{u_r}{r} \tag{1.19}$$

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The Duhamel–Neumann constitutive relation is given as

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} - 3K\alpha T \tag{1.20}$$

The listed stress components are therefore

$$\sigma_{rr} = 2\mu\varepsilon_{rr} + \lambda\ (\varepsilon_{kk}) - 3K\alpha T \tag{1.21}$$

$$\sigma_{\theta\theta} = 2\mu\varepsilon_{\theta\theta} + \lambda\ (\varepsilon) - 3K\alpha T \tag{1.22}$$

$$\sigma_{r\theta} = 2\mu\varepsilon_{r\theta} \tag{1.23}$$

Here, λ and μ are the Lamé constants, K is the bulk modulus, and δ_{ij} is the Kronecker delta.

VI. SPECIAL CASE

Consider the prescribed interior temperature distribution

$$f(\theta,\ t) = \left(1 - e^{-t}\right) cos\theta \tag{1.24}$$

Since $cos\theta = P_1(cos\theta)$, this input contains only the $n = 1$ Legendre mode. The reconstructed outer temperature therefore takes the form

$$g(\theta,\ t) = cos\theta\ \mathrm{L}^{-1}\left\{\left[\frac{j_1(qa)}{j_1(q\xi)}\right]\ \frac{1}{s(1\ +\ s)}\right\} \tag{1.25}$$

This particular case shows how a simple angular dependence reduces the reconstruction to a single modal contribution. Numerical evaluation requires specified values of a, ξ, and k, together with a stated inverse-Laplace procedure.

VII. NUMERICAL ILLUSTRATIONS AND GRAPHICAL REPRESENTATION

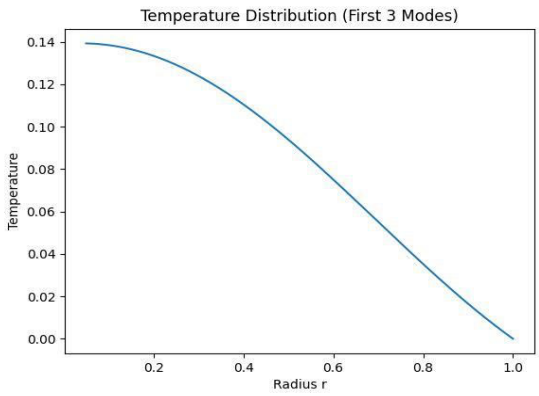

Fig. 1 Temperature distribution

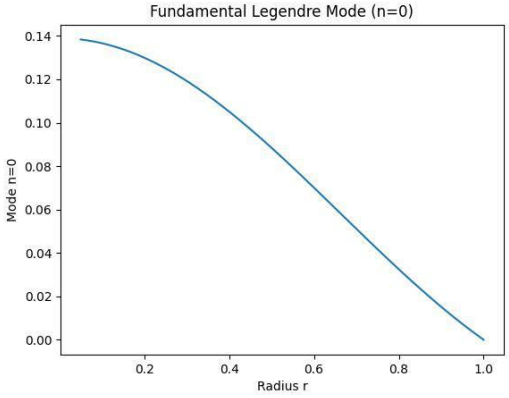

Fig. 2 Fundamental Legendre mode

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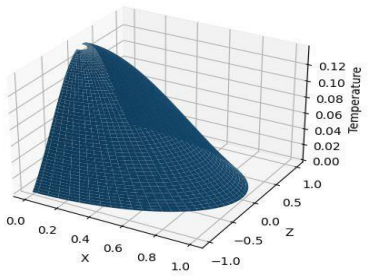

Fig. 3 Three-dimensional surface representation

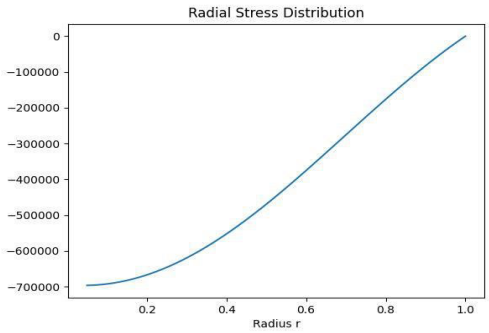

Fig. 6 Radial stress distribution

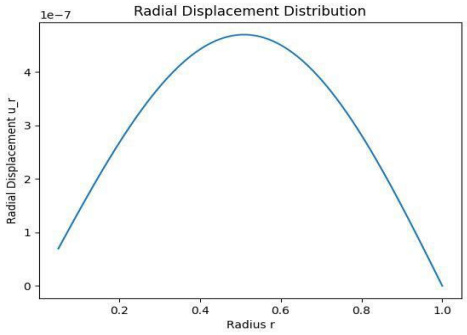

Fig. 4 Radial displacement distribution

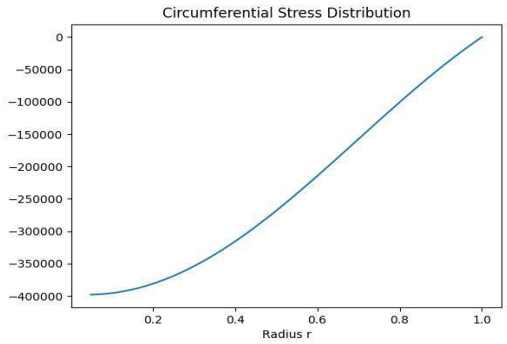

Fig. 7 Circumferential stress distribution

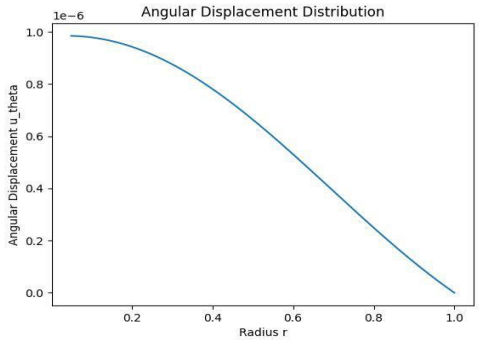

Fig. 5 Angular displacement distribution

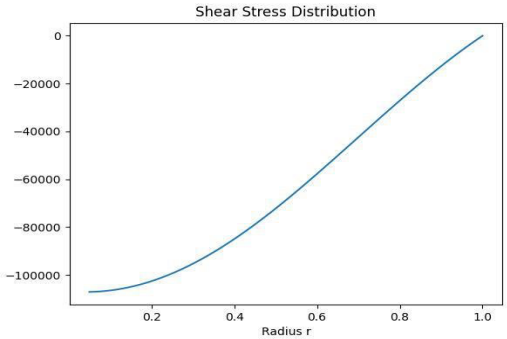

Fig. 8 Shear stress distribution

VIII. APPLICATIONS AND FUTURE SCOPE

The formulation may be relevant to thermal-boundary reconstruction and stress assessment in spherical components. Potential application areas include turbine-rotor, thermal analysis, temperature reconstruction in nuclear fuel pellets, spherical pressure vessels, and aerospace thermal-protection systems. These are prospective applications.

Further work could examine functionally graded materials, fractional heat-conduction models, non-Fourier thermoelasticity, and experimental validation of the two-dimensional solid-sphere model. Future studies should report material parameters, boundary conditions, numerical methods, and validation measures to make the results reproducible.

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IX. CONCLUSION

A modal framework has been presented for a two-dimensional, axisymmetric inverse transient thermoelastic problem in a homogeneous isotropic solid sphere. Expanding the temperature field in Legendre polynomials reduces the governing heat equation to radial-time equations for individual modes. In the Laplace domain, the ratio of spherical Bessel functions relates the known interior-temperature coefficients to the unknown outer-boundary coefficients. The prescribed input $f(\theta, t) = (1 - e^{-t})\ cos\theta$ illustrates the single-mode case. The manuscript also records displacement and stress relations and includes illustrative plots.

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